



A Formal Model for Verification of Dynamic Consistency of KBSs

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Abstract—This paper proposes a translation of the main concepts involved in Knowledge Based Systems Verification into a theoretical metalanguage based on Halmos and Leblanc's "Monadic and Polyadic Algebras." These algebras are expressed in terms of a few basic concepts of preorder-category theory.

Any Knowledge Base (KB) may be considered as a set of arrows that gives rise to a preorder category C , called "the N -category associated to the KB." Two subsets are defined in C : $E_q = \{x : x \rightarrow q \text{ is an arrow in } C\}$, and $E^p = \{x : p \rightarrow x \text{ is an arrow in } C\}$, where q and p , respectively, are a goal and a conjunction of facts.

A logic is a pair (C, E^p) ; it is consistent if it is not the case that both a proposition and its negation are simultaneously in E^p , which is equivalent to the inequality $E^p \neq C$. In this context, the two main ideas in the paper are the following. First, to characterize forward reasoning consistency of a KB with respect to a set of facts, in terms of consistency in (C, E^p) , where C is the N -category associated to the KB, and p is the conjunction of all the facts in the given set. Second, to characterize the absence of conflict in backward reasoning in terms of some relations among sets of the form E^p and E_q , for appropriate p and q .

The aim of the paper is to present ideas for further discussion leading to the construction of formal logico-algebraic models for verification, rather than to propose a completed model. It presents just one possible approach which may suggest others.

Keywords—Formal models for KBS's, Formal approach to verification, Algebraic logic.

1. INTRODUCTION

The aim of the present paper is to propose a translation of the concepts of forward and backward reasoning consistency of Knowledge Based Systems (KBSs) into a theoretical metalanguage based on Halmos [1] and Leblanc's [2] "Monadic and Polyadic Algebras." A few papers dealing with the

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same topic are quoted in the bibliography [3–6] but their techniques are different than the ones we propose here: the same could be said of other methods that we do not mention for the sake of brevity. Nazareth and Kennedy [4] base their study on relations in graphs. Meseguer [3] uses Petri nets. Nguyen *et al.* [5] classifies the KBSs in subsets of rules and establishes a structured comparison among rules in each module. Rousset [6] rewrites the rules of the KBS to detect pairs of conclusions that form a restriction, in order to find counterexamples for the hypothesis of consistency of the KBS. Our approach consists in providing an algebraic structure for KBSs.

The theoretical metalanguage to be developed in this paper is based on the version of Halmos' algebras called "*N*-categories" [7]; this last paper will hereafter be quoted as "*N*-categories." The reason for using the language of preorder categories instead of that of algebras is not an essential one, but there are some advantages, such as that the existence of isomorphic objects makes it unnecessary to build the Lindenbaum algebra, or that the unification power of the language of categories simplifies and interrelates concepts. The treatment is just Boolean because the model developed here is intended to be as elementary as possible. We wish to stress that our aim is not to provide algorithms for verification, but rather to suggest a theoretical framework for expressing KBSs verification concepts, as some basis for discussion for future building of better verification models.

Section 2 deals, first, with the concept of *N*-category, and second with a reinterpretation of consistency in propositional calculus by using some special subsets, denoted E^p and E_q , of an *N*-category. Sections 3 and 4, respectively, deal with the application of *N*-category concepts to the study of consistency in propositional KBSs and in KBSs formed with rules using predicates in one variable, as, for instance, $P(x) \wedge \neg Q(x) \rightarrow R(x)$. Section 5 deals shortly with rules using predicates in several variables. Section 6 contains an extension of the theory that provides additional consistency criteria. In Section 7 some applications of the theory are discussed.

Before going into the topic of the paper, let us briefly describe some of the main concepts to be dealt with: (1) rules and facts (2) KB-arrow and C-arrow, (3) forward and backward reasoning, (4) "firing" a rule (5) constraint, (6) consistency in forward reasoning, and (7) backward reasoning conflict.

- (1) In this paper, a *rule* is a formula of the form $\sigma_1 \wedge \sigma_2 \wedge \cdots \wedge \sigma_m \rightarrow \sigma_n$, where σ_i ($i = 1, \dots, m, n$) are literals (propositions preceded or not by the symbol \neg in Section 3, and atomic formulas, preceded or not by the symbol \neg in Sections 4 and 5.) A set of rules is denoted as "a KB." The literals in the antecedent and the consequent of a rule are respectively termed as "IF and THEN elements." *Facts* will be defined in Section 3.2.
- (2) Let C be the *N*-category associated to a KB. C will be defined in Section 3.1, but we advance here an intuitive description. The objects of C are, in addition to 0 and 1, all the literals appearing in the rules of the KBS plus their *N*-categorical combinations, that is, combinations of conjunctions, disjunctions, and negations (for instance, if α and β are literals in the rules, then $\alpha \wedge \beta$, $\neg\alpha$, $(\alpha \wedge \beta) \vee \neg\alpha$, $\neg((\alpha \wedge \beta) \vee \neg\alpha)$, etc, are objects of C). In the language of categories, \wedge , \vee , and \neg will be, respectively, written as a product $\langle \cdot, \cdot \rangle$, a coproduct $[\cdot, \cdot]$, and a functor N . We call *KB-arrows* the rules explicitly given by the KB. We call *C-arrows* all the other arrows in C . Intuitively, the C-arrows are the implications that could be derived in classical logic from the KB-rules, if these are considered as the axioms of a theory. For instance, if $\alpha \rightarrow \beta$ and $\gamma \rightarrow \delta$ are KB-arrows, then $\alpha \wedge \gamma \rightarrow \beta \wedge \delta$, $\alpha \vee \gamma \rightarrow \beta \vee \delta$, are C-arrows, but $\alpha \vee \gamma \rightarrow \beta \wedge \delta$ is not. If $\alpha \rightarrow \beta$ is a KB-arrow then $\neg\beta \rightarrow \neg\alpha$ is a C-arrow.
- (3) *Forward reasoning* consist of the repeated application of the logical rule of *modus ponens* and the logical rule of and-introduction, "from σ_i and σ_j obtain $\sigma_i \wedge \sigma_j$," to a set of well formed formulas. Let us, for instance, consider the formulae: $\sigma_1 \wedge \sigma_2 \rightarrow \sigma_3$, $\sigma_4 \rightarrow \sigma_5$, $\sigma_3 \wedge \sigma_5 \wedge \sigma_6 \rightarrow \sigma_7$, $\sigma_1 \wedge \sigma_2$, and σ_4 . The formulae σ_3 and σ_5 are obtained by *modus ponens* and the formula $\sigma_3 \wedge \sigma_5$ by and-introduction; from this last formula and the classical logic

tautology $A \wedge B \rightarrow C$ iff $A \rightarrow (B \rightarrow C)$, $\sigma_6 \rightarrow \sigma_7$ is obtained by *modus ponens*. For a given KB, *backward reasoning* from a formula, usually termed as a “goal,” is any procedure for either finding a set of facts, such that the goal can be obtained by forward reasoning from the KB and that set of facts, or proving that no such set exists. For instance, in order to find a goal σ_6 from the rules $\sigma_1 \wedge \sigma_2 \rightarrow \sigma_3$, $\sigma_4 \rightarrow \sigma_6$, and $\sigma_3 \wedge \sigma_5 \rightarrow \sigma_6$, the following argument ought to be pursued. Finding σ_6 requires having either σ_4 or both σ_3 and σ_5 . On its side, finding σ_3 requires having both σ_1 and σ_2 . Then, if either σ_4 or $\sigma_1, \sigma_2, \sigma_5$, are given as facts, σ_6 can be obtained from them.

- (4) Given a set of facts, we say that an arrow is *fireable* if, first, it is either a KB-arrow or the result of applying contravariance (from $a \rightarrow b$ obtain $\neg b \rightarrow \neg a$) to a KB-arrow, and, second, its antecedent can be obtained from the KB and the given set of facts by a forward reasoning with no C-arrows other than those obtained by contravariance. The result of the firing of the arrow is its consequent, and we will say that a formula is *reachable* from a set of facts if it is the result of firing some rules for that set of facts.
- (5) A *constraint* is a formula of the form $A \rightarrow \text{False}$, for A a conjunction. It is a restriction given by an expert in the sense that he/she assesses that, for instance, the conditions α and β can never occur together, denoted as $\alpha \wedge \beta \rightarrow \text{False}$. They should be added to the KB as new rules. Consequently, in the N -category associated to the KB, they are introduced as KB-arrows, in our example, as $\alpha \wedge \beta \rightarrow 0$, 0 being the initial object of the N -category.
- (6) A KB is *consistent in forward reasoning with respect to a given set of facts* if no two formulae of the form α , $\neg\alpha$, or α, β for $\alpha \wedge \beta \rightarrow \text{False}$ a constraint, can be obtained by forward reasoning from the KB and the given set of facts.
- (7) *Backward reasoning conflict* is a question of inadequacy rather than of logical inconsistency. Given a goal q and a set Γ of facts, the conjunction of which is p , backward reasoning conflict takes place in the following two cases. First: the goal is not reachable by forward reasoning from the facts in Γ . Second: there are at least two sequences of literals or conjunctions of literals, found by application of forward reasoning to Γ and to the arrows of the N -category associated to the KB, that begin in p , end in q and, respectively, contain two literals (or conjunctions of literals) which are contradictory, that is, either G is $\neg H$ or $G \wedge H \rightarrow \text{False}$. This is similar to proving some mathematical theorem by means of proofs that are at conflict with each other.

The main idea in the paper is the characterization of both forward reasoning consistency and absence of conflict in backward reasoning, in terms of some N -categorical relations that use sets E^p and E_q , for appropriate p and q . For the sake of brevity, most proofs of the propositions in this paper have been omitted.

2. N -CATEGORIES

2.1. Definition

An N -category C is a preorder category, together with a contravariant functor $N : C \rightarrow C$, such that:

- (i) C has a terminal object 1 ;
- (ii) C has finite coproducts $[\cdot, \cdot]$;
- (iii) the functor N^2 is naturally equivalent to the identity in C , i.e., $N^2 a \cong a$ for any object a in C ; and
- (iv) $a \rightarrow b$ is an arrow in C iff $[Na, b] \cong 1$, for any two objects $a, b \in C$.

2.2. Remark

Any N -category C has the following properties: (i') C has an initial object $0 \cong N 1$, (ii') C has finite products $\langle \cdot, \cdot \rangle$, defined as $\langle a, b \rangle = N[N a, N b]$ (De Morgan's Law), (iii') C is distributive, i.e., $\langle [a, b], [a, c] \rangle \cong [a, \langle b, c \rangle]$. C has pseudocomplements; c is a pseudocomplement of a relative to b if the following property holds: "for any x in C , $x \rightarrow c$ is an arrow in C iff $\langle a, x \rangle \rightarrow b$ is an arrow in C ." It is straightforward to check that $[N a, b]$ is a pseudocomplement of a relative to b , unique up to isomorphisms.

2.3. Explanatory Notes

In this subsection, just some intuitive comments are given. We refer the reader to [7] for a formal presentation of N -categories.

- (i) If a and b are formulae, it is natural to propose the following list as a dictionary to translate propositions to objects of N -categories. Let a and b be two propositions: they may be, respectively, identified with two objects a and b . We write: $a \vee b = [a, b]$, $a \wedge b = \langle a, b \rangle$, $\neg a = N a$, T (truth) = 1, F (falsity) = 0. Nevertheless $a \rightarrow b$ is an arrow, not an object. As in logic, implication is translated to $\neg a \vee b = [N a, b]$, the pseudocomplement of a relative to b , we may rewrite $[N a, b]$ as $a \Rightarrow b$, so that (iv) of the definition in 2.1, changes to " $a \rightarrow b$ is an arrow in C iff $a \Rightarrow b \cong 1$."
- (ii) This translation allows us to identify objects in C and individual propositions. Remember that in Boolean algebras the elements of the algebra are whole classes of equivalent propositions. In C , equivalent propositions correspond to isomorphic objects.
- (iii) The justification for using N -categories is the following.
 - (iii.1) Arrows correspond to implications.
 - (iii.2) It is known that a coproduct of two objects a and b is characterized in preorders as an object $[a, b]$ together with arrows $a \rightarrow [a, b]$ and $b \rightarrow [a, b]$, such that if there are arrows $a \rightarrow c$ and $b \rightarrow c$, there exists just one arrow $[a, b] \rightarrow c$ that makes the diagram formed by all these mentioned arrows to be commutative: but this is precisely the characterization of the disjunction $a \vee b$. Similarly, the product $\langle a, b \rangle$ characterizes $a \wedge b$.
 - (iii.3) The third condition in the Definition in 2.1 characterizes negation because $N^2 a \cong a$ is $\neg\neg a = a$, and the contravariance of N makes $a \rightarrow b$ hold iff $N b \rightarrow N a$ holds.
 - (iii.4) The objects 1 and 0, respectively, represent truth and falseness.
 - (iii.5) The condition " $a \rightarrow b$ is an arrow in C iff $a \Rightarrow b \cong 1$ " translates " a implies b iff $\neg a \vee b$ is true."
- (iv) There may be logical formulae involving implications as $(a \rightarrow b) \rightarrow (c \rightarrow d)$. These correspond to arrows in N -categories, between objects involving the symbols N , $\langle \cdot, \cdot \rangle$, $[\cdot, \cdot]$, \rightarrow , and \Rightarrow . The symbol \rightarrow corresponds to the main implication of the corresponding formula: in our example, the one that stands between the two parentheses. The other implication arrows denote pseudocomplements, which are objects. In our example, the implication above can be written in terms of N -categories as $(a \Rightarrow b) \rightarrow (c \Rightarrow d)$. In Section 7, arrows like $(a \Rightarrow b)$ will be called " \Rightarrow -arrows."

2.4. An Expression of Consistency in Propositional Calculus

2.4.1. The sets E_q and E^p

For q , an object of an N -category C , let E_q be the set $\{x : x \rightarrow q \text{ is an arrow in } C\}$ whose arrows are the same as in C . Defining $N_q : E_q \rightarrow E_q$ by $N_q x = \langle N x, q \rangle$, it results that N_q is a contravariant functor; so that the pair (E_q, N_q) becomes an N -category with q as terminal object and the same coproducts as in C . The sets $E^p = \{x : p \rightarrow x \text{ is an arrow in } C\}$ are defined similarly. The definition of E^p allows that this collection may in general be interpreted as the

probable propositions of a (finitely axiomatized) theory, p being the conjunction of the axioms of the theory. The elements the disjunction of which is q (which we may call co-axioms), can be considered to generate E_q in a dual form as the axioms of a theory generate the provable propositions.

2.4.2. Definitions

An N -categorical interpretation of a logic of propositions is a pair (C, E^p) which describes the set of propositions, together with those which are provable.

A logic (C, E^p) is consistent if it is not the case that both a proposition and its negation are simultaneously provable, which is equivalent to the inequality $E^p \neq C$, that is, iff $p \neq 0$.

A logic (C, E^p) is complete if for any proposition a we have that either $a \in E^p$ or $Na \in E^p$, but not both. This is equivalent to the maximality of E^p (E^p is maximal if p is minimal, regarding the ordering induced in C by the preorder character.)

3. A STUDY OF PROPOSITIONAL KBSs

3.1. Introduction

An N -category C contains a KB if the literals appearing in the rules of the KB are a subset of its set of objects, the rules of the KB are a subset of its set of arrows and, for each constraint $A \rightarrow \text{False}$ in the KB, there is an arrow of the form $A \rightarrow 0$ in C , for 0 the initial object.

DEFINITION. The N -category associated to a KB is the smallest N -category containing such a KB.

In the remainder of the section, C will denote this N -category, and the sets E^p and E_q are referred to C . The initial “0” is the product of all IF elements which are not THEN elements, together with all the negations of the THEN elements which are not IF elements. The final object “1” is dually defined. The objects of C are, in addition to 0 and 1, all the literals appearing in the rules of the KB plus their N -categorical combinations, that is, combinations of products, coproducts, and applications of the functor N (see Concept (2) of Section 1).

In this section, the following propositional logic knowledge base will be used as illustration or particular case, suitable for straightforward generalization:

$$\begin{array}{llll}
 A \vee B \vee C \rightarrow D & D \wedge J \rightarrow I & X \rightarrow H & D1 \wedge E1 \rightarrow F1, \\
 T \wedge L \rightarrow I & \neg D \rightarrow R & A \rightarrow X & V1 \wedge C1 \wedge U1 \rightarrow F1, \\
 D \vee E \rightarrow H & J \wedge K \rightarrow H & \neg G \rightarrow Q & A1 \wedge B1 \rightarrow C1, \\
 H \wedge \neg F \wedge \neg G \rightarrow I & J \wedge K \rightarrow G & \neg F \rightarrow S & V \wedge \neg O \wedge \neg P \rightarrow Q, \\
 F \wedge G \rightarrow M & \neg E \rightarrow M & V \wedge U \rightarrow S & R \vee Q \vee S \rightarrow T, \\
 I \vee G \vee M \rightarrow V & V \wedge W \rightarrow \neg D & G1 \wedge H1 \rightarrow C1. &
 \end{array}$$

The rules containing disjunctions in the antecedent should be changed to sets of rules that have a single element in the antecedent. For instance, $A \vee B \vee C \rightarrow D$ changes to the set of three rules: $A \rightarrow D$, $B \rightarrow D$, $C \rightarrow D$.

As an illustration of what KB-arrows, C-arrows, and objects are in the N -category associated to the above KB, let us consider just two rules, $V \wedge U \rightarrow S$ and $A \rightarrow D$. They are KB-arrows. From the objects V , U , S , A , and D , some of the new objects that can be formed with them, and some of the C-arrows between these objects are shown in Figure 1. The initial object is: $0 = \langle V, U, A, NS, ND \rangle$.

3.3.3. Important remark

If the proposition in Section 3.3.2 holds for all maximal consistent subsets of FB, the KB is said to be forward reasoning consistent with respect to its FB.

3.3.4. A second criterium for consistency using completeness

Let p be the product of the elements of any consistent subset of FB, and let $C \neq E^p$. The smallest N -category of C containing both E^p and E_{N_p} is $C' = E^p \cup E_{N_p}$. The following properties hold:

- (i) $1 \in E^p$, $0 \notin E^p$, $1 \notin E_{N_p}$, $0 \in E_{N_p}$;
- (ii) $E^p \cap E_{N_p} = \emptyset$, because if $\alpha \in E^p \cap E_{N_p}$, then $p \rightarrow \alpha \rightarrow N_p$, and so $E^p = C$; and
- (iii) E^p and E_{N_p} are maximal in C' because if there exists $q \in C'$, $q \not\cong 0$, such that $E^p \subset E^q$, then as $q \notin E^p$, $q \in E_{N_p}$ and then both $q \rightarrow p$ and $q \rightarrow N_p$ are arrows in C' : this means $q \cong 0$, a contradiction.

If E^p is maximal in C' , then (C', E^p) is complete. The argument just pursued justifies the following proposition.

PROPOSITION. Let C' , Γ , and p be as above; (C, E^p) is forward reasoning consistent with respect to Γ iff (C', E^p) is complete.

3.4. No-Conflict Backward Reasoning

For the sake of clarity, the argument in this section will be carried on by using the KB in Section 3.1 as an example, but it is generalizable to any KB.

Let us first classify the KB-rules into *modules*, where each module is the set of all chainings of KB-rules which end in the same THEN element. This is shown in Figure 2.

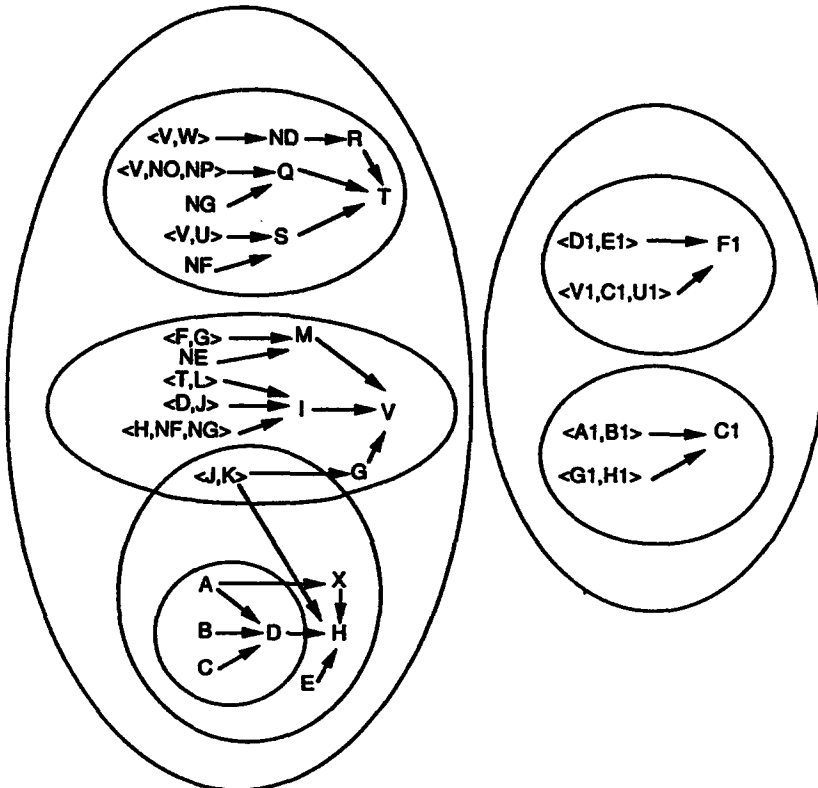


Figure 2.

The resulting modules must be assigned to different collections, so that each two of these collections do not have common objects nor some common product. In our example, one collection consists of the modules ending in H , V , T , and another collection is the one formed by the modules ending in $F1$ and $C1$. Note that inside a module there can be submodules, and also that a proposition or conjunction of propositions may be in two or more modules.

An order of precedence among the modules of each collection must be established as follows. The module ending in, say, H is to be considered before the module ending in, say, V if the second one has an antecedent which contains H as one of its conjuncts. For instance, the first antecedent in $H \wedge \neg F \wedge \neg G \rightarrow I \rightarrow V$ contains H . It may happen that the mentioned precedence criterium holds reciprocally for two modules, as it is the case for the modules ending in V and T , because we have both $V \wedge W \rightarrow \dots \rightarrow T$ and $T \wedge L \rightarrow \dots \rightarrow V$. In this case, the one ending in V is to be chosen because the prior selection of the one ending in H makes it necessary to consider as second the one ending in V .

Note that in more complex collections of modules, several orderings could be obtained following the above precedence criterium; but the idea is to establish just one of them, and this is enough for our purpose.

For α ranging over H , V , T , let E_α be the smallest N -category containing the module of all arrows that end in α . By extension, we also call these E_α "modules."

Suppose that T is proposed as a goal for the KB in Section 3.1. Let us form the module E_T , and let the other modules in the collection to which E_T belongs, be ordered according to the order described above. In our example, we obtain the ordering E_H , E_V , E_T .

Suppose also that we are given a consistent set of facts $\Gamma = \{A, NF, NG, U\}$. The following strategy is proposed.

- (i) Form $E^{p_1} \cap E_H$, where p_1 is the product of the elements of the set Γ , and E_H is the first module in our ordering. If H is reachable from Γ , then $E^{p_1} \cap E_H \neq \emptyset$. Let us see with an example that the converse also holds. Suppose we have a chain of C-arrows or KB-arrows in $E^{p_1} \cap E_H \neq \emptyset$ like, for instance, $p_1 \rightarrow A \rightarrow \langle D, X \rangle \rightarrow H$. Having the C-arrow $A \rightarrow \langle D, X \rangle$ means that $A \rightarrow D$ and $A \rightarrow X$ are KB-arrows. As A is a fact, they can both be fired, resulting in D and X . Consequently, any arrow with antecedents $\langle D, X \rangle$, D , or X is fireable if it is a KB-arrow. Both the arrows $\langle D, X \rangle \rightarrow X$ and $\langle D, X \rangle \rightarrow D$ are C-arrows, therefore nonfireable. Now, we have $\langle D, X \rangle \rightarrow H$ in our chain: $\langle D, X \rangle \rightarrow H$ is not a KB-arrow and then, at least one of $X \rightarrow H$ and $D \rightarrow H$ is a KB-arrow, in which case, at least one of them is fireable (in this case, both are), leading to H as the final result.
- (ii) Form $E^{p_2} \cap E_V$, for p_2 the product of the elements of $\Gamma \cup \{H\}$. V is reachable from $\Gamma \cup \{H\}$ iff $E^{p_2} \cap E_V \neq \emptyset$.
- (iii) Form $E^{p_3} \cap E_T$, for p_3 being the product of the elements of $\Gamma \cup \{H, V\}$. T is reachable from $\Gamma \cup \{H, V\}$ iff $E^{p_3} \cap E_T \neq \emptyset$.

Thus the inequalities $E^{p_1} \cap E_H \neq \emptyset$, $E^{p_2} \cap E_V \neq \emptyset$, and $E^{p_3} \cap E_T \neq \emptyset$, translate the facts that H , V , and T are, respectively, reachable from Γ , $\Gamma \cup \{H\}$, and $\Gamma \cup \{H, V\}$. If all these three inequalities hold (as it is the case in our example), T is reachable from Γ .

Regarding the relations between E^p and E_q in a backward reasoning, the following possibilities may appear:

$$E^p = E_q = C, \quad (1)$$

$$E^p = C, \quad E_q \neq C, \quad (2)$$

$$E_q = C, \quad E^p \neq C, \quad (3)$$

$$E^p \neq C, \quad E_q \neq C. \quad (4)$$

The definition of conflict in backward reasoning given in concept (7) of Section 1 can be translated

to the assertion that conflict in backward reasoning takes place only when Equation (1) holds. This justifies the following criterium.

If, in addition to the reachability conditions above, the following inequalities hold: $E^{p_1} \cap E_H \neq C$, $E^{p_2} \cap E_V \neq C$, and $E^{p_3} \cap E_T \neq C$, there is no-conflict backward reasoning in the KB presented as example, for the goal T and the given set Γ .

4. MONADIC KBSs

4.1. Monadic Predicate Logic KBSs

4.1.1. Introduction

A monadic predicate logic KB (to be hereafter called “monadic KB”) consists of rules $\sigma_1 \wedge \sigma_2 \wedge \dots \wedge \sigma_m \rightarrow \sigma_n$, where σ_i ($i = 1, \dots, m, n$) are atomic formulae, like Ax , Bx , etc., preceded or not by the symbol \neg . These rules refer to a set X , in the sense that the variable x in Ax , Bx , etc., ranges over the elements of X . X may be finite or infinite, even though in expert systems it would in most cases be finite.

Let KB' be a knowledge base which consists of all the X -instances of all the rules of our monadic KB. It is clear that KB' is a propositional logic knowledge base, and we will refer to it as the “propositional KB corresponding to the monadic KB”; if X is finite, this KB is also finite.

C will denote all along this section the N -category associated to the *propositional* KB corresponding to a monadic KB. It is for this C and for the set X above to which all notions in Section 7 of “ N -categories,” to be outlined below, refer. The construct C^X , to be described next, will be “the N -category associated to the *monadic* KB.”

For C and X as above, we define a collection of functions C^X as follows. Let P be a predicate of our KB, $f_P : X \rightarrow C$ is the function that sends $x \in X$ to $Px \in C$. We make the convention of symbolically identifying each of the propositional functions f_P with the corresponding predicate P . Thus, we will write P , Q , etc, instead of f_P , f_Q , etc.

C^X , with the arrows defined as “ $P \rightarrow Q$ is an arrow in C^X iff $Px \rightarrow Qx$ are arrows in C , for all $x \in X$,” is a preorder category. This category, together with the functor $N^X : C^X \rightarrow C^X$ given by $N^X Px = N Px$ is an N -category. Its terminal object is the propositional function $1^X \in C^X$ given by $1^X x = 1 \in C$; the coproduct is defined as $[P, Q]^X x = [Px, Qx]$. Note that C can be identified with the subset of C^X consisting of all constant functions of C^X .

Let us define quantifiers. Let \exists be a function $\exists : C^X \rightarrow C^X$ defined as $\exists P = \bigcup \text{rg}(P)$ (coproduct of the objects in the range of P). If one sees C as the set of constant functions of C^X , \exists is a function $C^X \rightarrow C$. It can be proved that:

- (i) $Px \rightarrow \exists P \in C$ for all $x \in X$, that is $P \rightarrow \exists P$ in C^X ;
- (ii) If $P \rightarrow Q$ in C^X , then $\exists P \rightarrow \exists Q$ in C (and also in C^X).

Thus \exists is functorial. By defining $\forall : C^X \rightarrow C^X$ by $\forall = N \exists N^X$, then a functor with dual properties to those of \exists is obtained.

4.1.2. Some relevant constructs in C^X

Let the sets E^{p_x} , E_{q_x} , $\langle E^p \rangle_X$, $\langle E_q \rangle_X$ be:

$$\begin{aligned} E^{p_x} &= \{P \in C^X \mid p \rightarrow Px \text{ is an arrow in } C \text{ for all } x \in X, \text{ and } p \text{ a constant function in } C^X\}, \\ E_{q_x} &= \{Q \in C^X \mid Qx \rightarrow q \text{ is an arrow in } C \text{ for all } x \in X, \text{ and } q \text{ a constant function in } C^X\}, \\ \langle E^p \rangle_X &= \{P \in C^X \mid px \rightarrow Px \text{ is an arrow in } C \text{ for all } x \in X, \text{ and } p \text{ a fixed element of } C^X\}, \\ \langle E_q \rangle_X &= \{Q \in C^X \mid Qx \rightarrow qx \text{ is an arrow in } C \text{ for all } x \in X, \text{ and } q \text{ a fixed element of } C^X\}, \end{aligned}$$

Note, from the definitions of E^{p_x} , E_{q_x} above and the definition of quantifiers, that $E^{p_x} = \{P \in C^X \mid p \rightarrow \forall P \text{ is an arrow in } C\}$, and $E_{q_x} = \{Q \in C^X \mid \exists Q \rightarrow q \text{ is an arrow}$

in C). Particular cases of the sets just defined are the sets E^{pX} , E_{qX} , $\langle E^p \rangle_X$, and $(E_q)_X$, where p is a product and q a coproduct. There is an arrow to $P \in C^X$ from the product $\langle P1, P2, \dots, Pn \rangle^X$ (resp., from $Q \in C^X$ to the coproduct $[Q1, Q2, \dots, Qm]^X$), if for all $x \in X$, $\langle P1, P2, \dots, Pn \rangle^X x \rightarrow Px$ (resp., $Qx \rightarrow [Q1, Q2, \dots, Qm]^X x$) is an arrow in C . Note also that $\langle P1, P2, \dots, Pn \rangle^X x = \langle P1x, P2x, \dots, Pnx \rangle = \langle p1, p2, \dots, pn \rangle$ is a product (resp., $[Q1, Q2, \dots, Qm]^X x = [Q1x, Q2x, \dots, Qmx] = [q1, q2, \dots, qm]$ is a coproduct) of elements of C . In this situation, we have the following.

- (i) E^{pX} (resp., E_{qX}) is the set of elements $P \in C^X$ (resp., $Q \in C^X$) to which there are arrows from a given product $p = \langle P1, P2, \dots, Pn \rangle^X$ (resp., from which there are arrows to a given coproduct $q = [Q1, Q2, \dots, Qm]^X$) of *constant* functions of the type $P_i : X \rightarrow C$, $i = 1, \dots, n$, (resp., $Q_j : X \rightarrow C$, $j = 1, \dots, m$) which each sends *all* $x \in X$ to an element $p_i \in C$ (resp., to an element $q_j \in C$). In this case, p is just one element $p = \langle p1, p2, \dots, pn \rangle$ of C , and q is just one element $q = [q1, q2, \dots, qm]$ of C .
- (ii) $\langle E^p \rangle_X$ (resp., $(E_q)_X$) is the set of elements $P \in C^X$ (resp., $Q \in C^X$) to which there are arrows from a given product $p = \langle P1, P2, \dots, Pn \rangle^X$ (resp., from which there are arrows to a given coproduct $q = [Q1, Q2, \dots, Qm]^X$) of functions $P_i : X \rightarrow C$, $i = 1, \dots, n$ (resp., $Q_j : X \rightarrow C$, $j = 1, \dots, m$) which *need not be constant* as in the previous case.

These last two sets, $\langle E^p \rangle_X$ and E^{pX} , will be used for the characterization of forward reasoning consistency from a conjunction (a product) p of a given set of facts.

4.1.3. Consistency and completeness

DEFINITION. A monadic logic is a pair $(C^X, \langle E^p \rangle_X)$. As for the propositional case, $(C^X, \langle E^p \rangle_X)$ is consistent (resp., complete) iff $\langle E^p \rangle_X$ is a proper subset of (resp., maximal in) C^X .

$(C^X, (E_q)_X)$ is also called “a logic” and it is defined to be consistent iff $C^X \neq (E_q)_X$. Similar definition of logic and its consistency hold for the particular cases E^{pX} and E_{qX} .

PROPOSITION.

- (i) (C^X, E^{pX}) is consistent (resp., complete) iff $(C^X \cap C, E^{pX} \cap C) = (C, E^p)$ is consistent (resp., (C, E^p) is complete).
- (ii) If $(C, E^{\forall p})$ is consistent (resp., complete), then $(C^X, \langle E^p \rangle_X)$ is consistent (resp., complete).

4.2. N-Categorical Monadic First Order Predicate Calculus Applied to KB Systems

The facts in a monadic KB, as for instance Ax , Bx , etc., for the monadic KB of Figure 3, will be called *global facts* (they actually are the objects A , B , etc., of C^X , but we will adopt, in the rest of the paper, the notation Ax , Bx , etc.). The facts in the propositional KB corresponding to the monadic KB are the X -instances Aa , Ba , etc., of the global facts, and they will be called *particular facts*. Global facts are then collections of particular facts, and particular facts are objects of C . The Fact Base FB, the consistent subset of FB, and the maximal consistent subset of FB are defined for both global and particular facts analogously as they were defined in Section 3.2.

Constraints may now be particular and global, having respectively the forms, $Pa \wedge Qa \wedge \dots \rightarrow 0$ for some constant element a in X , and $Px \wedge Qx \wedge \dots \rightarrow 0$ for x ranging over the elements of X ; P , Q , etc., may be preceded by the negation symbol.

4.3. Forward Reasoning Consistency

4.3.1. First criteria

PROPOSITION. Let C^X be the N -category associated to a monadic KB, and let p be the product of the elements of a consistent subset Γ of the set of global facts. If $(C, E^{\forall p})$ is consistent, then

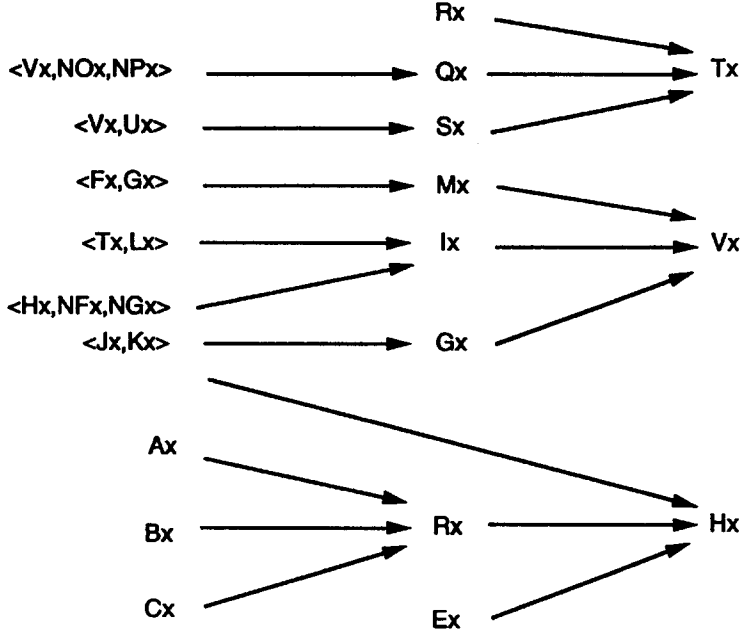


Figure 3.

$(C^X, \langle E^p \rangle_X)$ is consistent, in which case the monadic KB is forward reasoning consistent with respect to Γ .

PROPOSITION.

- (i) Let C^X be the N -category associated to a monadic KB. Given the propositional KB corresponding to the monadic KB, let p be the product of the elements of a consistent subset Δ of the set of particular facts. (C^X, E^{pX}) is consistent iff (C, E^p) is consistent.
- (ii) If (C^X, E^{pX}) is consistent, that is, if (C, E^p) is consistent, then the propositional KB corresponding to the monadic KB is forward reasoning consistent with respect to Δ .

4.3.2. A second criterium of consistency using completeness

Let C^X be the N -category associated to a monadic KB, and let p be the product of the elements of a consistent subset Γ of the set of global facts. $C'^X = \langle E^p \rangle_X \cup (E_{N_p})_X$ is an N -category which is a subcategory of C^X . As for the propositional case, the following result holds.

PROPOSITION. Let C'^X , Γ and p be as above. $(C^X, \langle E^p \rangle_X)$ is forward reasoning consistent with respect to Γ iff $(C'^X, \langle E^p \rangle_X)$ is complete.

Let C^X be as above. Given the propositional KB corresponding to a monadic KB, and a consistent subset Δ of its set of particular facts, let p be the product of the elements of Γ . Let $C'^X = E^{pX} \cup E_{N_p, X}$ and let $C' = C'^X \cap C : E^{pX}$ is maximal in C'^X iff E^p is maximal in C' . The proof is as follows. Suppose that E^p is maximal and that there exists $p' \not\approx 0$ such that $E^{pX} \subset E^{p'X} \neq C'^X$, that is $p' \rightarrow p \in C$, and so $E^p \subset E^{p'} \neq C'$ which contradicts the maximality of E^p : then E^{pX} is maximal in C'^X . The proof of the converse is similar. Thus, we have the following proposition.

PROPOSITION. (C^X, E^{pX}) is forward reasoning consistent with respect to Δ iff (C', E^p) is complete.

4.4. No-Conflict Backward Reasoning

As for the propositional case, our argument in this section uses a particular example, the KB in Figure 3, but its results are generalizable to any KB. An ordering among the modules is established following the same criteria as in Section 3.4. This produces the ordering

E_{Hx} , E_{Vx} , E_{Tx} . Similarly to Section 3.4, if the goal is Tx , and a consistent set of global facts is $\Gamma = \{Ax, NFx, Gx, Ux\}$, the criterium given by the following two propositions holds.

PROPOSITION. *Let p_1 , p_2 , and p_3 , respectively, be the products of the objects in Γ , the objects in $\Gamma \cup \{Hx\}$, and the objects in $\Gamma \cup \{Hx, Vx\}$. Hx is reachable from Γ iff $\langle E^{p_1} \rangle_X \cap (E_{Hx})_X \neq \emptyset$. Vx is reachable from $\Gamma \cup \{Hx\}$ iff $\langle E^{p_2} \rangle_X \cap (E_{Vx})_X \neq \emptyset$, and Tx is reachable from $\Gamma \cup \{Hx, Vx\}$ iff $\langle E^{p_3} \rangle_X \cap (E_{Tx})_X \neq \emptyset$. If these three inequalities hold (as it is the case in our example), Tx is reachable from Γ .*

PROPOSITION. *If, in addition to the conditions in the proposition above, the following inequalities hold: $\langle E^{p_1} \rangle_X \cap (E_H)_X \neq C^X$, $\langle E^{p_2} \rangle_X \cap (E_V)_X \neq C^X$, $\langle E^{p_3} \rangle_X \cap (E_T)_X \neq C^X$, then there is no-conflict backward reasoning for the goal Tx and the given Γ .*

A similar result holds for particular facts and goals.

5. POLYADIC FIRST ORDER PREDICATE CALCULUS APPLIED TO KBSs

In order not to make this paper unnecessarily long, the theory of consistency for rules of KBSs using polyadic predicates will not be developed because it is totally analogous to the monadic case. Such a theory is based in “ N -categories,” (Section 8) with a small change which is briefly described next. Consider a KB involving predicates in several variables: In “ N -categories,” as in Halmos’ original study, the propositional functions of several variables are treated as functions $X^I \rightarrow C$, where I is an arbitrary set of indices, X a set, and C an N -category. We have finite Cartesian products, instead of X^I , for interpretations of KBSs. These functions should be total and of the form, say, $P : X \times Y \times Z \times W \rightarrow C$, where $X \times Y \times Z \times W$ is the set of the tuples the KB is referring to, and C is the N -category associated to the propositional KB corresponding to the given polyadic KB. The study parallels the monadic case, by considering sets $(E_q)_{X \times Y \times Z \times W}$, $E_{qX \times Y \times Z \times W}$, $\langle E^p \rangle_{X \times Y \times Z \times W}$, and $E_{X \times Y \times Z \times W}^p$.

6. FURTHER RESULTS

6.1. Introductory Note

The argument in this section holds for the propositional, the monadic, and the polyadic cases. We just write C , E^p , and E_q , under the understanding that in the monadic and polyadic cases C is C^X , E^p is E^{pX} or $\langle E^p \rangle_X$, etc., p is

6.2. Definitions

- (i) An object $a \in C$ is a unity when, for any set E^p , if $a \in E^p$, then $E^p = C$.
- (ii) A finite set of objects in C is a unitary set when for any set E^p that contains it, $E^p = C$.

6.3. Remarks

- (i) C contains always unitary sets (for instance, $\{a, Na\}$). The unitary sets are the expression of contradiction in C .
- (ii) In addition to the unitary sets provided by logical considerations, there are others derived from constraints. For instance, a constraint of the type $\alpha \wedge \beta \rightarrow \text{False}$ gives rise to the unitary set $\{\alpha, \beta\}$.

6.4. Proposition

$$E_{\langle a_1, a_2, \dots, a_n \rangle} = \cap E_{a_i}, \quad i = 1, \dots, n.$$

6.5. Proposition

If $\{a_1, a_2, \dots, a_n\}$ is a unitary set, then $E^p \neq C$ iff $p \notin \cap E_{a_i}$ for $i = 1, \dots, n$.

6.6. Additional Consistency Criteria

6.6.1. Proposition

A third criterium for consistency in forward reasoning: *If for a given set of facts whose product is p , E^p does not contain any unitary set, then (C, E^p) is consistent in forward reasoning with respect to that set of facts.*

PROOF. C contains at least an unitary set. As this set is not in E^p , $E^p \neq C$, and thus (C, E^p) is consistent in forward reasoning. ■

Generalizing, *if for all conjunctions p in maximal consistent sets, E^p does not contain any unitary set, then (C, E^p) is consistent.*

6.6.2. Proposition

A fourth criterium for consistency in forward reasoning: *Let Π be the set of all products of the objects in each maximal consistent sets of facts. Let $\{a_1, \dots, a_n\}$ be any unitary set; (C, E^p) is consistent in forward reasoning iff $(\cap_{i=1, \dots, n} E_{a_i}) \cap \Pi = \emptyset$.*

PROOF. If such an intersection is empty, in particular p does not belong to it. Then $E^p \neq C$ by the proposition in Section 6.5 and thus, (C, E^p) is consistent in forward firing. The converse also holds because the proposition in Section 6.5 expresses a condition of the type *iff*. ■

7. SOME TENTATIVE APPLICATIONS OF THE THEORY

7.1. Introduction

The aim of this paper was to propose a formal mathematical model. As the construction of KBSs verification tools is a very difficult task for the specialists working on the field, we cannot at all claim that, from the ideas in this paper, a tool could be obtained. Nevertheless, some procedures based on the model can be proposed, so that they may complement procedures obtained by other means. This is the topic of the present section.

7.2. A First Approach Based on the Proposition in Section 6.6.2

- (i) Suppose that a maximal consistent set of facts L has been given. Let p be the product of the elements in L .
- (ii) If an object α belongs to $\cap E_{a_i}$, a_i ($i = 1, \dots, k$) being the objects that form a unitary set, then $\alpha \rightarrow \langle a_i, a_i \dots, a_i \rangle$ is an arrow.
- (iii) If $p \rightarrow \alpha$, then $p \in \cap E_{a_i}$, so by the proposition in Section 6.6.2 E^p is not proper, and then, there is inconsistency in forward firing with respect to the facts in L .

This suggest the following procedure:

- (1) *Determine the unitary sets.* These may arise from constraints as, for instance, by declaring α_1 and α_2 to be incompatible. They may also be of many other forms derived from logic, but in this case, the relevant ones arise from pairs $(\gamma, N\gamma)$, for γ an object that may be a combination of products and coproducts.
- (2) *Characterize the objects α belonging to $\cap E_{a_i}$, $i = 1, \dots, k$.* Let us illustrate this characterization with an example: to find arrows $\alpha \rightarrow \langle \alpha_1, \alpha_2 \rangle$, for $\{\alpha_1, \alpha_2\}$ a unitary set. In order to do it, first, we should collect the C-arrows having α_1 and α_2 among their THEN elements. They can be of the forms:
 - (2.i) $\neg\alpha_1 \wedge A1 \rightarrow A2$, $\neg\alpha_2 \wedge B1 \rightarrow B2$,

- (2.ii) $A3 \wedge A4 \rightarrow \alpha_1$, $B3 \wedge B4 \rightarrow \alpha_2$,
 where $A1$, $A3$, $A4$, $B1$, $B3$, and $B4$, are literals or conjunctions of literals, and $A2$ and $B2$, are literals. The two arrows in (2.i) change to $A1 \wedge \neg A2 \rightarrow \alpha_1$ and $B1 \wedge \neg B2 \rightarrow \alpha_2$.
- (3) In order to find whether some object or objects belong to E^p , we look for THEN elements appearing in C-arrows that have some element of the set L in their antecedents. This search is made easier if the appropriate arrows are changed in the following way. From the set of facts in L and *modus ponens*, several of the components of the IF elements of some (it may be none) of the C-rules in part (2) may be eliminated. They result in objects involving, in addition to the symbols N , $\langle \cdot, \cdot \rangle$, $[\cdot, \cdot]$, the symbol \Rightarrow . All these objects belong to E^p and have α_1 or α_2 as the right term of the arrows \Rightarrow . From these \Rightarrow -arrows, other \Rightarrow -arrows (remember that they are objects) having $\langle \alpha_1, \alpha_2 \rangle$ as their right term may be obtained. These objects, if there are any, also belong to E^p .
- (4) Now, an external source, the opinion of an expert, is needed to check consistency. If the expert judges that, at least for one of the \Rightarrow -arrows found as described in (3), the antecedent follows from p , there is inconsistency in forward reasoning. This expert's judgment must be given with a degree of certainty.

Let us see an example. Suppose that we are given the following KB:

$$\begin{aligned} R1. \langle A, NB, F \rangle &\rightarrow H, & R3. \langle G, E \rangle &\rightarrow NQ, \\ R2. \langle D, H \rangle &\rightarrow L, & R4. \langle P, ND \rangle &\rightarrow Q. \end{aligned}$$

Suppose also we are given the set of facts $L = \{A, ND, E\}$ and an unitary set $\{B, Q\}$. The rules above change successively to:

$$\begin{aligned} \text{(i)} \quad R'1. \langle A, F, NH \rangle &\rightarrow B, & R'3. \langle G, E \rangle &\rightarrow NQ, \\ R'2. \langle D, H \rangle &\rightarrow L, & R'4. \langle P, ND \rangle &\rightarrow Q; \end{aligned}$$

and to:

$$\begin{aligned} \text{(ii)} \quad R''1. A &\rightarrow (F \Rightarrow (NH \Rightarrow B)), & R''3. E &\rightarrow (G \Rightarrow NQ), \\ R''2. \langle D, H \rangle &\rightarrow L, & R''4. ND &\rightarrow (P \Rightarrow Q). \end{aligned}$$

If p is the product of the objects of L , there results, by examining C , that the following \Rightarrow -arrows, are in E^p ; this fact corresponds to a forward firing, that is, the application of *modus ponens* to L and the rules $R''1$, $R''3$, and $R''4$.

$$1. (F \Rightarrow (NH \Rightarrow B)), \quad 3. (G \Rightarrow NQ), \quad 4. (P \Rightarrow Q).$$

The next thing to do is, when possible, to obtain the information that can be extracted by N -category arguments from 1, 3, and 4, as for instance the existence of a new constraint $\langle G, P \rangle \Rightarrow \langle Q, NQ \rangle \cong 0$, or that $\langle F, NH, P \rangle \rightarrow \langle B, Q \rangle$. The final step, is to ask the expert about what degree of certainty (see Section 7.4 below) can be assigned to the assessment that there is a C-arrow from the coproduct of some of the facts in L to $\langle F, NH, P \rangle$ or $\langle G, P \rangle$.

7.3. A Second Approach Based on the Proposition in Section 3.3.4

In Section 3.3.4, $E^p \cup E_{N_p}$ is a proper part of the N -category C , for p a product of the objects in a consistent set, say L , of facts. If $\emptyset \neq E^p \cup E_{N_p}$, then:

- (i) For any $\alpha \in C$, either $\alpha \in E^p$ or $N\alpha \in E^p$ (if $\alpha \notin E^p$) then $N\alpha \in E^p$ because, otherwise, $\alpha, N\alpha \in E_{N_p}$, which implies that $N_p \cong 1$, that is $p \cong 0$. This would mean that the set L is an inconsistent set of facts. The same argument holds for a pair α_1, α_2 , forming a constraint.
- (ii) If $[\delta, \gamma] \in E^p$, then $\delta \in E^p$ or $\gamma \in E^p$ or both because, if $\delta \notin E^p$, $\gamma \notin E^p$, then $N\delta, N\gamma \in E^p$, and thus, $\langle N\delta, N\gamma \rangle \cong N[\delta, \gamma] \in E^p$ which contradicts $[\delta, \gamma] \in E^p$.

This argument suggest the following procedure. Suppose we have the following KB:

$$\begin{aligned} R1. \langle A, B, F \rangle \rightarrow H, \quad R3. \langle H, E \rangle \rightarrow P, \\ R2. \langle D, H \rangle \rightarrow L, \quad R4. \langle H, ND \rangle \rightarrow Q. \end{aligned}$$

Suppose we are given the set of facts $L\{A, F, ND, E\}$, and the unitary set $\{P, Q\}$. From L , we obtain by N -category considerations (that is, by using *modus ponens* and/or the logical axioms for implication):

$$\begin{aligned} 1'. \quad [NB, H], \quad 3'. \quad [NH, P], \\ 2'. \quad [ND, NH, L], \quad 4'. \quad [NH, Q]. \end{aligned}$$

All these coproducts are in E^p , for p the conjunction of the elements in L . According to the criterium stated in (ii) above, at least one disjunct in each of $1'$ to $4'$ must be in E^p . As for the procedure in Section 7.2, an external factor must operate at this step. Suppose that an expert assesses, with some degree of certainty, that $H \in E^p$. Then $NH \notin E^p$. This fact, together with $3'$ and $4'$, implies that both $Q \in E^p$, $P \in E^p$, which is against $E^p \cup E_{N_p}$ being a proper subset of C . Then the supposition $H \in E^p$, yields conflict. It can be checked that if the expert says that $ND \in E^p$, there is no conflict.

7.4. A Final Comment to this Section

Two procedures that follow from the theory have been suggested. At the present time we are just starting to check how they, or others, such as finding if there are common objects to E^p and E_{N_p} , can be applicable to medium size KBs. Some steps are mechanical like for instance, finding coproducts as in Section 7.3. Other steps require to express in mathematical terms, the certainty of external items, such as the expert's opinions: for this, we are trying the translation of certainty in terms of Rasiowa's regular logic [8]. In any case, the implementation of the ideas presented in this section requires a study of complexity and uncertainty, that we have not done yet.

8. CONCLUSIONS

The consideration of a KB as forming the core of logico-algebraic constructs (not necessarily N -categories, which are just an example), leads to the construction of structures that provide theoretical frameworks for relations among concepts involved in KBS's construction. Such relations are in this way expressed in terms of a theoretical metalanguage, which may act as a specification language for expressing conditions of verification.

The implementation of the model is a very difficult task, but at least some procedures that could be improved introducing uncertainty at some step of the search for inconsistency, can be suggested by the theory. Ours has been just a succinct presentation of these procedures. Much more study of their complexity and uncertainty has yet to be done. At the present time, we are trying to extend the model to structures more general than N -categories, considering the KB as the set of axioms of a theory, under nonclassical logics.

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